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Alternans and Spiral Breakup in a Modified FitzHugh-Nagumo Model of Cardiac Cell Dynamics

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In this talk, we introduce a new two-variable partial differential equations model of electrical wave propagation of cardiac cells. We modify the FitzHugh-Nagumo equations [1, 2] without changing slow manifold instead changing velocity of each branches of slow manifold. The nullclines of our model are shown in Figure 1(a). Both nullclines intersect each other on the left branch of the u-nullcline only once at a point, which is called the rest state of the excitable media. We investigate numerically the existence of traveling wave solutions of the proposed model of reaction-diffusion system of equations. We study the instability of the periodic traveling wave solutions in one-dimensional simulation. When the parameter b is gradually decreased, the traveling wave loses its stability via a supercritical Andronov-Hopf bifurcation (see Fig. 1(b)). In two dimensions, the emergence of stable spiral wave pattern is observe in the proposed model, which occurs when the heart is malfunctioning (i.e. ventricular tachycardia). The oscillation of spiral pulse width i.e. alternans is observe in a specific parameter regime. In addition, we show that unstructured spiral breakup running to complex spatio-temporal pattern occurs as a direct consequence of this instability of traveling wave solution. This chaotic behavior of the medium is called ventricular fibrillation. Moreover, we study the widths of the spiral pulses numerically in order to understand the underlying mechanisms of the complex patterns.

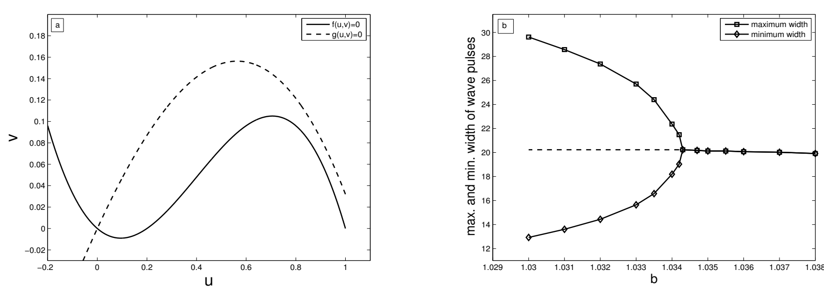

Figure 1: (a) The nullclines and (b) the bifurcation diagram of the proposed model.

References

[1] R. FitzHugh, *Impulse and physiological states in theoretical models of nerve membrane*, Biophys. J. **1**, pp. 445-465 (1961).

[2] J. S. Nagumo, S. Arimoto and S. Yoshizawa, *An active pulse transmission line simulating nerve axon*, in *Proc. IRE. 50*, pp. 2061-2071(1962).